# Coordination on Use of Non-deferred Electronic Payment Instrument

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**Abstract**

In the use of electronic payment technology, there is strategic complementarity and hence room for self-fulfilling multiple equilibria. But existing relevant literature is silent about how agents' expectations become coordinated. This paper resolves the coordination problem in the use of a non-deferred electronic means of payment which can be represented by a debit card. We focus on that because it is almost a perfect substitute for cash. The presence of exogenous shocks which have a fundament impact on the cost of the technology turns out to make agents coordinate their expectations in a particular way. We also show that a higher inflation and distortionary financing scheme for the cost of debit-card transactions disturb the coordination in the use of debit cards.

*Keywords*: cash, coordination, electronic payment, strategic complementarities.
*JEL classification*: E40, E60

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# 1. Introduction

Recently electronic payment transactions have increased dramatically. In particular, the growing trend of debit-card transactions is remarkable. Borzekowski, Kiser and Ahmed (2008) report that since 1996, debit-card transactions have grown at an average rate of more than 20 percent per year. As a consequence, debit cards have become one of the primary means of electronic payment for in-store purchases these days (see, for example, Schuh and Stavins 2012).

Such a rapid increase in the use of debit cards has drawn economist's attention on the choice of means of payment, particularly between fiat money and debit cards. (See, for instance, He, Huang and Wright 2008, Klee 2008, Kim and Lee 2010, Li 2011, Schuh and Stavins 2012, and Lotz and Vasseli 2013.) However, these works are silent to the scale-of-economy effect in the use of debit cards. An economy of scale is indeed one of the key features of an electronic payment technology. Relative to cash, debit cards in practice are a technology with substantial fixed cost (setting up clearing and verification systems, issuing cards, etc.) but very low per-transaction marginal cost. This then implies that the benefit from debit-card transactions will be enhanced with agents' willingness to use because it can eventually lower per transaction cost.

By taking into account the scale-of-economy effect in use of debit-card payment, Huang, Kim and Lee (2014) examine an optimal allocation scheme for its cost in a search-theoretic environment. However, they do not discuss much about the coordination issue in connection with the use of debit cards. As mentioned, there are strategic complementarities in the use of debit cards in the sense that the payoff for each debit-card user relies on actions of other agents. Therefore, as in typical models with strategic complementarities, this model exhibits multiple equilibria: an agent's belief will not be uniquely determined according to the state of economy and self-fulfilling prophecies would lead to either an equilibrium with all cash

transactions or one with all debit-card transactions. We then should be able to say about how agents' expectations become coordinated and which equilibrium would be played.

In this paper, we try to provide an answer to this coordination problem by introducing aggregate shock on the state of economy as in Frankel and Pauzner (2000), Burdzy, Frankel and Pauzner (2001), and Araujo and Guimaraes (2014), for instance. Our model is essentially similar to Huang, Kim and Lee (2014) and focuses on a debit card because as the representative non-deferred electronic means of payment (i.e., the point of delivering good and the point of clearing the trade are coincident), it is almost a perfect substitute for cash. But the cost structures of debit-card transactions are different in a way that it is constant over time in their model, while it changes randomly in our model. Specifically, we introduce a shock to the state of economy which has a fundamental impact on the cost of the technology such as the development of information technology. We then explore an interaction between the state of economy and agents' willingness to coordinate in the use of debit cards. In doing so, we consider the following 2 cases: one with friction in switching means of payment and the other with small friction in changing means of payment (limiting case of the former).

We first show that in an economy with the friction, there is a unique equilibrium in which agents are always willing to coordinate in the use of debit cards if currently realized shock on the state of economy is greater than a threshold level which is characterized as a function of the fraction of existing debit-card users. That is, agents' expectations are determined uniquely according to the state of economy which captures the key components of debt-card cost and any other variables which are irrelevant to agents' payoffs such as sunspot do not play any role. We then show that in the limiting economy where agents are free to switch means of payment, the uniqueness result is preserved but has a different feature in the sense that the threshold level does not depend on the fraction of existing debit-card users. This implies that even though the friction on switching means of payment is small enough, the presence of an exogenous shock on the primitives of electronic payment technology can

resolve an indeterminacy problem.

We also show that agents are less willing to coordinate in the use of debit cards as inflation goes up. It is in stark contrast to Monnet and Roberds (2008) where inflation can stimulate the use of credit cards. This difference is mainly attributed to the fact that a credit card is a deferred payment instrument, whereas a debit card is a non-deferred payment one. In our model, the quantity of good traded via a debit card is greater than that in cash and hence, a debit-card trader demands more money to make such a relatively large transaction. This suggests that a debit-card trader is more susceptible to inflation and consequently, coordinate in the use of debit cards is discouraged in an inflationary economy. In a similar vein, distortionary financing scheme for the cost of debit-card transaction turns out to disturb the coordination on the use of debit cards.

## 2. Model

The background environment is a version of Lagos and Wright (2005) with uncertainty about the user cost of electronic means of payment which, like a debit card, is a non-deferred one and does not have credit function. Time is continuous and goes on forever but trading opportunities occur in a sequence of periods at times $t = 0, \tau, 2\tau, \cdots$. Some trades take place in a centralized market and others in a decentralized market with bilateral random matching. There are many varieties of special goods and a general good. The general good can be produced and traded in the centralized market, whereas the special goods can be produced and traded in the decentralized market. All goods are divisible, perishable and have to be consumed right after production. There is another object, called money which is perfectly divisible, durable and evolution of total stock is controlled by the government.

The economy is populated with a $[0, 1]$ continuum of infinitely lived agents and each of whom can consume a subset of specialized goods. Each agent can transform labor into

one of the special goods which cannot be consumed by himself. The utility from $q$ units of special-good consumption is given by $u(q)$ where $u'' < 0 < u'$. The disutility from producing $q$ units of a special good is given by $c(q)$ where $c' > 0$ and $c'' \geq 0$. All agents can consume and produce the general good. The utility obtained from the net consumption $x$ of general good is simply $x\tau$ which implies that producing the general good for oneself is worthless. Agents discount future utility at the same rate $e^{-\rho\tau}$ with $\rho > 0$. There is no discounting within a period $\tau$.

In the decentralized market, each agent is randomly matched with another agent and he becomes either a buyer or a seller with an equal probability $\alpha\tau \in (0, 1)$. In a bilateral meeting, trading histories are private and agents cannot commit to future actions, which rules out any possibility of credit trades and hence a medium of exchange is essential. (See, for example, Kocherlakota 1998, Wallace 2001, Corbae, Temzelideset and Wright 2003, and Aliprantis, Camera and Puzzello 2007.) That is, transfer of money in the form of either cash or debit card should be made in exchange for goods produced.

In a bilateral meeting, a buyer makes take-it-or-leave-it offer $(q, p)$ to a seller where $q$ denotes quantity of goods produced by a seller for a buyer and $p$ denotes the amount of money transferred by a buyer to a seller. A buyer can pay $p$ in cash (hereinafter C-payment) at the disutility cost of $\eta\phi p$ where $\phi$ denote the value of money in terms of general good. This cost can be interpreted as capturing the inconvenience of carrying cash around, the risk of loss or theft and the foregone interest (see, for instance, Baumol 1952, Tobin 1956, Humphrey 2004, He, Huang and Wright 2008, and Monnet and Roberds 2008).

A buyer can also transfer $p$ via a debit card (hereinafter E-payment): i.e., E-payment system can transfer money from a buyer's account to a seller's account directly. E-payment transactions incur the society a fixed cost $\Omega(z)\tau$ in terms of general good and its users pay the relevant cost in the upcoming centralized market by producing general goods where $\Omega'(z) <$

0.[1] The parameter $z$ denotes the state of economy which captures the key components of E-payment cost such as the development of information technology. As in Burdzy, Frankel and Pauzner (2001), $z$ follows a random walk such that either $z_{t+\tau} = z_t + \nu\tau + \sigma\sqrt{\tau}$ or $z_t + \nu\tau - \sigma\sqrt{\tau}$ with equal probability.[2] Notice that the dynamics of $z_t$ can be characterized by a variance $\sigma^2$ and a trend $\nu$: $\sigma^2$ measures the size of the random component, whereas $\nu$ captures the deterministic part of $z$. We then assume that C-payment is the dominant payment for $z < -\bar{z}$ and E-payment is the dominant payment for $z > \bar{z}$. Throughout, we assume $\bar{z}$ is large enough so that the dominant regions are very remote each other.

The cost $\Omega(z)$ is financed by imposing fee $\omega$ on each E-payment where $\omega$ should satisfy

$$\Omega(z) = \mathbf{S}\left[\theta\omega + (1-\theta)\omega\right]. \tag{1}$$

Here $\mathbf{S}$ denotes the instantaneous measure of E-payment transactions, $\theta \in [0,1]$ is the share of social cost allocated to a buyer, and $(1-\theta) = \breve{\theta} \in [0,1]$ is that to a seller. It is worthwhile to note that as shown in Huang, Kim and Lee (2014), $\theta$ can be interpreted as taxation on buyer's labor and has a lump-sum feature in the sense that it does not affect quantity consumed in exchange for money transferred via a debit card. Meanwhile, $\breve{\theta}$ can be interpreted as taxation on buyer's consumption and is distortionary in the sense that it affects quantity consumed in the debit-card transactions. The E-payment system executes only the order of transferring money instantaneously and hence credit trade is not feasible. We further assume that for the case of $\mathbf{S} < \gamma$, $\omega$ is given by $\omega = \Omega(z)/\gamma$, which can be regarded as the government subsidy to the E-payment system if there is too few E-payments where $\gamma$ is very small so that the economy of scale ($\omega$ declines as $\mathbf{S}$ increases) is valid almost everywhere. Notice that

[1] As in Hayashi and Keeton (2012) and Huang, Kim and Lee (2014), $\Omega$ can be interpreted as the "social cost" of using electronic payment system.

[2] In the environment with constant $z$, multiple equilibria exist: either all agents choose E-payment, or all agents choose C-payment. That is, for a given initial condition, all agents might use E-payment if they believe that all other agents do so, whereas all agents might choose C-payment if they believe all other agents will not use E-payment.

imposing fee on no-traders or cash traders is not feasible because such transactions are not recorded in the system and cost is collected not on the spot but in the centralized market.

Finally, the economy has friction such that in each period, a measure $k\tau$ of agents is randomly selected and given opportunities to switch their means of payment for the upcoming bilateral trades. Putting differently, it is costless to change means of payment for some fraction of agents, whereas it is very costly for the remains. The realization of this opportunity is independent across time and agents. In section 4, we will also consider its limiting case (small friction) by letting $\tau \to 0$ and then $k \to \infty$.

In sum, the sequence of events in a period of each $\tau$ is as follows. First, a state of an economy ($z$) and preference shock (buyer or seller in the decentralized market) are realized. Then agents enter the decentralized market: a buyer moves into with cash or a debit card depending on her choice of means of payment, whereas a seller can keep her money at home at no cost. Trade occurs if agreement is reached between a buyer and a seller. After bilateral trades but before entering into the centralized market, the opportunity to switch the means of payment for the following period is realized. On arriving in the centralized market, each agent receives lump-sum transfer of money from the government and then trades general good where unlike the decentralized market, E-payment is not available.

# 3. Equilibrium

We now formulate an equilibrium in a recursive manner and work backward from the centralized market to the decentralized market in a generic period $\tau$.

## 3.1. Bellman Equations

We let $W_t^o(m, z)$ be the value function pertaining to the beginning of $t$-period centralized market where an agent holds $m$ units of money and is stuck in payment system $o \in \{E, C\}$

with the realized state $z$. We let $V_t^o(m,z)$ be the value function pertaining to the beginning of $t$-period decentralized market where an agent as a buyer holds $m$ units of money and is stuck in payment system $o \in \{E, C\}$ with the realized state $z$. Let also $\phi_t$ be the price of money in terms of general good in the $t$-period centralized market and $b_t$ be the lump-sum transfer of money, by which the government controls the inflation rate $\mu_t = \frac{\phi_t - \phi_{t+\tau}}{\phi_{t+\tau}\tau}$.

Noting that the opportunity of switching means of payment arrives before opening the centralized market, $W_t^o$ and $V_{t+\tau}^o$ share the same $o$-system. Then the centralized-market problem for an agent entering with $m$ is given by

$$W_t^o(m,z) = \max_{x,m',o'} \left\{ \begin{array}{l} x\tau + e^{-\rho\tau}\mathbb{E}_z[\alpha\tau V_{t+\tau}^o(m',z') \\ +(1-\alpha\tau)(k\tau W_{t+\tau}^{o'}(m',z') + (1-k\tau)W_{t+\tau}^o(m',z'))] \end{array} \right\} \tag{2}$$

$$\text{s.t. } x\tau = \phi_t(m+b_t) - \phi_t m'$$

where the expectation $\mathbb{E}_z$ is taken over the distribution of the following $z'$ conditional on current $z$. Notice that $o'$ is contingent on the realization of next period state $z'$, while $m'$ has to be chosen before the realization of $z'$. Here we use the fact that under the buyer-take-all bargaining rule, the payoff of a seller in the decentralized market is equivalent to that of a no-trader. Substituting $x\tau$ from the constraint, we have

$$\begin{aligned} W_t^o(m,z) &= \phi_t(m+b_t) \\ &\quad +e^{-\rho\tau}\max_{m'}\left\{\alpha\tau\left[\mathbb{E}_z(V_{t+\tau}^o(m',z')) - \phi_{t+\tau}m'\right] + (\phi_{t+\tau} - e^{\rho\tau}\phi_t)m'\right\} \\ &\quad +e^{-\rho\tau}(1-\alpha\tau)k\tau\mathbb{E}_z\left\{\max_{o'}\left[W_{t+\tau}^{o'}(0,z')\right]\right\} \\ &\quad +e^{-\rho\tau}(1-\alpha\tau)(1-k\tau)\mathbb{E}_z\left[W_{t+\tau}^o(0,z')\right]. \end{aligned} \tag{3}$$

As in Lagos and Wright (2005), (3) implies that $W_t^o(m,z) = \phi_t m + W_t^o(0,z)$: i.e., regardless of $o \in \{E, C\}$, $W_t^o(m,z)$ is linear in money holdings ($m$). Notice that $[\mathbb{E}_z(V_{t+\tau}^o(m',z')) -$

$\phi_{t+\tau}m']$ is the net benefit of a buyer relative to a seller. Noting also that the choice of a means of payment is contingent on realization of $z'$, expectation operator ($\mathbb{E}_z$) comes before maximization operator ($\max_{o'}$). The sequence $\{\phi_t\}$ is controlled by the government and thus it is independent of realization $z'$.

Given the buyer-take-all trading protocol, $V_t^o(m,z)$ should satisfy

$$\begin{aligned} V_t^o(m,z) &= \max_{q,p,o'}[u(q) - \xi^o + k\tau W_t^{o'}(m-p,z) + (1-k\tau)W_t^o(m-p,z)] \\ &= \max_{q,p,o'}[u(q) - \phi_t p - \xi^o + \phi_t m + k\tau W_t^{o'}(0,z) + (1-k\tau)W_t^o(0,z)] \end{aligned} \tag{4}$$

subject to a seller's participation constraint [$c(q) \leq \phi_t p$ for $o = C$ and $c(q) \leq \phi_t p - \breve{\theta}\omega$ for $o = E$] and the transaction-cost function $\xi^o$ takes

$$\xi^o = \begin{cases} \theta\omega & \text{if} \quad o = E \\ \eta\phi_t p & \text{if} \quad o = C. \end{cases}$$

In addition, the take-it-or-leave-it offer $(q,p)$ for $o = C$ should satisfy

$$\max_{(q,p)}[u(q) - (1+\eta)\phi_t p] \tag{5}$$

$$\text{s.t. } c(q) = \phi_t p \ \text{ and } \ p \leq m$$

and that for $o = E$ should satisfy

$$\max_{(q,d)}[u(q) - \phi_t p] - \theta\omega \tag{6}$$

$$\text{s.t. } c(q) = \phi_t p - \breve{\theta}\omega \ \text{ and } \ p \leq m.$$

The solutions for (5) is given by

$$q^C = \begin{cases} \hat{q} & \text{if} \quad \phi_t m \geq \hat{q} \\ c^{-1}(\phi_t m) & \text{otherwise} \end{cases}$$

$$p^C = \begin{cases} \hat{q}/\phi_t & \text{if} \quad \phi_t m \geq \hat{q} \\ m & \text{otherwise} \end{cases}$$

where $u'(\hat{q}) = (1+\eta)c'(\hat{q})$. The solutions for (6) is given by

$$q^E = \begin{cases} q^* & \text{if} \quad \phi_t m \geq c(q^*) + \breve{\theta}\omega \\ c^{-1}(\phi_t m - \breve{\theta}\omega) & \text{otherwise} \end{cases}$$

$$p^E = \begin{cases} [c(q^*) + \breve{\theta}\omega]\phi_t^{-1} & \text{if} \quad \phi_t m \geq c(q^*) + \breve{\theta}\omega \\ m & \text{otherwise} \end{cases}$$

where $u'(q^*) = c'(q^*)$. Notice that C-payment cost $\eta$ is distortionary in the sense that the first best allocation ($q^*$) can never be chosen in C-payment.

Now from (4), $V_m^C(m,z)$, the marginal value of money for a buyer in the decentralized market with $o = C$ can be obtained as follows:

$$V_m^C(m,z) = \begin{cases} \phi_t & \text{if} \quad \phi_t m \geq \hat{q} \\ \frac{u'}{c'}|_{\phi_t m}\phi_t - \eta\phi_t & \text{otherwise} \end{cases}$$

and that with $o = E$, $V_m^E(m,z)$, is given by

$$V_m^E(m,z) = \begin{cases} \phi_t & \text{if} \quad \phi_t m \geq c(q^*) + \breve{\theta}\omega \\ \frac{u'}{c'}|_{\phi_t m - \breve{\theta}\omega}\phi_t & \text{otherwise.} \end{cases} .$$

Finally, (3) implies that the money demand for the next decentralized market ($m'$) is irrele-

vant to the current money holding ($m$) and solves

$$\max_{m'}\{\alpha\tau[\mathbb{E}_z(V_t^o(m',z')) - \phi_t m'] + (\phi_t - e^{\rho\tau}\phi_{t-\tau})m'\}. \tag{7}$$

The first order condition for $o = C$ is then

$$\alpha\tau\left(\frac{u'}{c'}|_{c^{-1}(\phi_t m')} - 1 - \eta\right) = e^{\rho\tau}(1 + \mu_{t-\tau}\tau) - 1 \tag{8}$$

and that for $o = E$ is

$$\alpha\tau\left(\mathbb{E}_z(\frac{u'}{c'}|_{c^{-1}(\phi_t m' - \breve{\theta}\omega)}) - 1\right) = e^{\rho\tau}(1 + \mu_{t-\tau}\tau) - 1. \tag{9}$$

The balance of money carried into next period is determined by the trade-off between inflation cost of holding money and expected benefit from a bilateral trade which differs across C-payment and E-payment. Notice that $\mu_{t-\tau}$ must be greater than zero so that the solution exists and for a given $\mu_{t-\tau} > 0$, the productions in (8) and (9) are uniquely determined respectively.

## 3.2. Iterative Conditional Dominance

Let $X_t$ be the fraction of agents who are locked into $E-$payment system after the choices are revised just before $t$-period centralized market. The public history at time $t$ is the evolution of the environment until $t - \tau$ such that $\{X_v, z_v, \phi_v\}_{v=0,\tau,\cdots,t-\tau}$ where the initial values of $X_0$ and $z_0$ are given. An agent's private history at time $t$ consists of her actions and the details of her matches through period $t$. An agent's information set at time $t = \tau, 2\tau, 3\tau, \cdots$ composes of the public history and her private history. Strategies are functions from the set of all information sets to the set of mixtures over $\{E, C\}$ which indicate what an agent will do if she receives an opportunity to change the means of payment. Notice that the effect

of money-holding distribution goes through the price ($\phi_v$) and hence its evolution is not included in the public history.

When an agent receives an opportunity to change the means of payment, she chooses the payment instrument given the probability distribution over paths $(z_{t+i\tau})_{i=0}^{\infty}$, the deterministic path $(\phi_{t+i\tau})_{i=0}^{\infty}$ which is controlled by the government, and her beliefs about path $(X_{t+i\tau})_{i=0}^{\infty}$ which will result from any given realization of $(z_{t+i\tau})_{i=0}^{\infty}$ and $(\phi_{t+i\tau})_{i=0}^{\infty}$. Notice that an agent readjusts her money holdings in the centralized market for the upcoming bilateral trade whether she receives a switching opportunity or not. If she is locked into $o_t \in \{E, C\}$, her expected payoff with respect to the individual shock is

$$\begin{aligned} W_t^{o_t}(m_t^{o_t}, z_t) &= \phi_t(m_t^{o_t} + b_t) \\ &\quad + e^{-\rho\tau}\left\{\alpha\tau\mathbb{E}_z\left[u(q_{t+\tau}^{o_t}) - \phi_{t+\tau}p_{t+\tau}^{o_t} - \xi_{t+\tau}^{o_t}\right] + (\phi_{t+\tau} - e^{\rho\tau}\phi_t)m_{t+\tau}^{o_t}\right\} \\ &\quad + e^{-\rho\tau}k\tau\mathbb{E}_z \max_{o_{t+\tau}}\left[W_{t+\tau}^{o_{t+\tau}}(0, z_{t+\tau})\right] + e^{-\rho\tau}(1-k\tau)\mathbb{E}_z\left[W_{t+\tau}^{o_t}(0, z_{t+\tau})\right] \end{aligned} \tag{10}$$

where $m_t^o$ and $m_{t+\tau}^{o_t}$ are the money carried into $t$-period centralized market and that carried into the decentralized market in the following period, respectively. [See Appendix for derivation of (10).] The term $\mathbb{E}_z\left[u(q_{t+\tau}^{o_t}) - \phi_{t+\tau}p_{t+\tau}^{o_t} - \xi_{t+\tau}^{o_t}\right]$ is the benefit from a bilateral trade as a buyer which will depend on $z_{t+\tau}$ through its effect on the cost sharing scheme (1) and trade specified in (6) if $o = E$. The term $(\phi_{t+\tau} - e^{\rho\tau}\phi_t)m_{t+\tau}^{o_t}$ is the change in the value of money holdings due to inflation.

Notice that if $z_t$ is realized, (7) uniquely determines the following period money holding $m_{t+\tau}$ which, together with the following period $\omega_{t+\tau}$, gives $(q_{t+\tau}^E, p_{t+\tau}^E)$. In (9), $(q_{t+\tau}^E, p_{t+\tau}^E)$ is contingent on the realization of $(z_t, z_{t+\tau})$ even though the expectation operator is taken as if $(q_{t+\tau}^E, p_{t+\tau}^E)$ depends on the whole sequence of realized $z_{t+i\tau}$. Then the relative payoff to choosing $E$-payment in the period $t$ $[\Delta(E, z_t) \equiv W_t^E(m_t^E, z_t) - W_t^C(m_t^C, z_t)]$ is given by[3]

[3]See Appendix for derivation of (11).

$$\Delta(E,z_t) = \sum_{i=1}^{\infty} e^{-\rho\tau i}(1-k\tau)_z^{i-1}\mathbb{E}_z \left\{ \begin{array}{c} \alpha\tau u(q^E_{t+i\tau}) + \left(1 - e^{\rho\tau t}\frac{\phi_{(i-1)\tau+t}}{\phi_{t+i\tau}} - \alpha\tau\right) c(q^E_{t+i\tau}) \\ + \left[\left(1 - e^{\rho\tau t}\frac{\phi_{(i-1)\tau+t}}{\phi_{t+i\tau}}\right)\breve{\theta} - \alpha\tau\right]\omega_{t+i\tau} \end{array} \right\} -$$

$$\sum_{i=1}^{\infty} e^{-\rho\tau i}(1-k\tau)^{i-1} \left\{ \begin{array}{c} \alpha\tau[u(q^C_{t+i\tau}) - \phi_{t+i\tau}p^C_{t+i\tau} - \eta\phi_{t+i\tau}p^C_{t+i\tau}] + \\ \left[\phi_{t+i\tau} - e^{\rho\tau t}\phi_{(i-1)\tau+t}\right] m^C_{t+i\tau} \end{array} \right\}. \tag{11}$$

With probability $(1-k\tau)^{i-1}$, she has no switching opportunities between $t$ and $t+i\tau$, and an agent chooses E-payment if the relative payoff in (11) is positive $[\Delta(E,z_t) > 0]$ and cash if it is negative $[\Delta(E,z_t) < 0]$. Notice that inflation cost is positive so that the money holdings is finite $(1 - e^{\rho\tau t}\frac{\phi_{(i-1)\tau+t}}{\phi_{t+i\tau}}) < 0)$. Notice also that a higher fraction of agents using E-payment implies a lower $\omega_{t+i\tau}$, a smaller restriction on production defined in (6) and hence a higher the relative payoff $\Delta(E,z_t)$. That is, in our model, strategic complementarities come from the economy of scale in the use of electronic payment system which implies the following result.

**Lemma 1 (Strategic complementarity)** *Consider the subsets of the space of states $(X,z)$, $\mathbb{S}_A$ and $\mathbb{S}_B$, and two associated strategies $A$ and $B$, respectively. In strategy $A$, agents receiving an opportunity of switching means of payment choose E-payment only in states $(X,z) \in \mathbb{S}_A$. In strategy $B$, agents choose E-payment only in states $(X,z) \in \mathbb{S}_B$. If $\mathbb{S}_A \subset \mathbb{S}_B$, then the relative payoff of choosing E-payment in case $B$ is at least as large as in case $A$.*

As in Frankel and Pauzner (2000), we now solve the model using a solution concept of the iterative elimination of conditionally dominated strategies. The process makes use of strategic complementarity implied in (1). Notice that due to our assumption, E-payment is a dominant choice if $z$ is sufficiently high, whereas C-payment is a dominant choice if $z$ is low enough. Although agents put most of the weight on payoffs that they receive very soon if the switching opportunities arrives frequently ($k$ large), as we will see below, the presence of these dominance regions still remain important because of backwards induction.

Let $Z_0(X)$ be the boundary of the region where an agent will choose E-payment regardless of the choices of other agents: i.e., even in the case in which she expects all other agents choosing after her to pick C-payment, she will choose E-payment. Since an agent know that other agents receiving an opportunity of switching a means of payment must choose E-payment to the right of $Z_0$, an agent actually wants to choose E-payment slightly to the left of $Z_0$ as well because of strategic complementarity. Therefore, there is a new boundary $Z_1$ to the left of $Z_0$. This process can be repeated infinitely which gives a sequence of cut-off functions $Z_2$, $Z_3$,... . Let $Z_\infty$ be the limiting cut-off function of this sequence. We then know that agents will choose E-payment when the current state is to the right of $Z_\infty$.

We now let $Z_0'$ be the boundary of the region where an agent will choose C-payment regardless of the choices of other agents: i.e., even in the case in which she expects all other agents choosing a means of payment after her to pick E-payment, she will choose C-payment. Exactly the same iteration above can be applied and we can obtain another limiting cut-off function $Z_\infty'$. These two limiting cut-off functions ($Z_\infty$ and $Z_\infty'$) coincide with each other as shown in Theorem 1 of Frankel and Pauzner (2000). This argument gives the following uniqueness result.[4]

**Proposition 1** *There exists a unique equilibrium in the model such that agents choose E-payment if and only if* $z \geq Z_\infty(X)$.

# 4. Small Friction Economy

In the previous section, we introduced the friction of switching the means of payment. We will now explore the case in which such friction shrinks to zero by letting $\tau \to 0$ and then $k \to \infty$ (i.e., $k\tau \to 0$). As shown in Appendix, we can first obtain the relative payoff to choosing $E$-payment when $\tau \to 0$ by taking limit to $\Delta(E, z_t)$ in (11):

[4]For more formal proof in an analogous environment, see Frankel and Pauzner (2000).

$$\lim_{\tau\to 0}\Delta(E,z_t) = \mathbb{E}_{z_t}\left\{\begin{array}{c}\int_0^\infty e^{-(k+\rho)v}\{\alpha[u(q^E_{t+v})-c(q^E_{t+v})-\omega_{t+v}] \\ -\mu_{t+v}c(q^E_{t+v})\}dv\end{array}\right\}$$
$$-\int_0^\infty e^{-(k+\rho)v}\left\{\begin{array}{c}\alpha[u(q^C_{t+v})-\phi_{t+v}p^C_{t+v}-\eta\phi_{t+v}p^C_{t+v}] \\ -\phi_{t+v}\mu_{t+v}m^C_{t+v}\end{array}\right\}dv. \quad (12)$$

Notice that as shown in Frankel and Pauzner (2000), the dynamics of $X$ as $k\to\infty$ bifurcates either upward (all agents use E-payment) or downward (all agents use C-payment). Starting with a state $(X^*, z^*)$, the chance of bifurcating up to $X=1$ converges to $1-X^*$, while the chance of bifurcating down to $X=0$ goes to $X^*$. The dynamics of $X$ can be approximated as $X^{\uparrow}_{t+v} = 1-(1-X^*)e^{-kv}$ with probability $(1-X^*)$ and $X^{\downarrow}_{t+v} = X^*e^{-kv}$ with probability $X^*$. In addition, the movement in $X$ is fast relative to the movement in $z$: in other words, $z$ can be considered constant relative to the movement of $X$. Then if we consider a constant inflation rate $\mu$, the agent's relative payoff to choosing $E$-payment can be approximated as follows:

$$\begin{aligned}\bar{\Delta}(E,z_t) &= (1-X^*)\left\{\int_0^\infty e^{-(k+\rho)v}\{\alpha[u(q^E_*)-c(q^E_*)-\omega^{\uparrow}_{t+v}]-\mu c(q^E_*)-\mu\breve{\theta}\omega^{\uparrow}_{t+v}\}dv\right\} \\ &\quad +X^*\left\{\int_0^\infty e^{-(k+\rho)v}\{\alpha[u(q^E_*)-c(q^E_*)-\omega^{\downarrow}_{t+v}]-\mu c(q^E_*)-\mu\breve{\theta}\omega^{\downarrow}_{t+v}\}dv\right\} \\ &\quad -\int_0^\infty e^{-(k+\rho)v}\{\alpha[u(q^C_*)-c(q^C_*)-\eta c(q^C_*)]-\mu c(q^C_*)\}dv \qquad (13)\end{aligned}$$

where $\omega^{\uparrow}_{t+v}$ ($\omega^{\downarrow}_{t+v}$) is the path associated with $X^{\uparrow}_{t+v}$ ($X^{\downarrow}_{t+v}$). In addition, noting $z_{t+\tau}=z_t$ when $\tau\to 0$, (8) and (9) with $\tau\to 0$ and constant $\mu$ imply $q^E_*$ and $q^C_*$ are constant, and satisfy the followings respectively:

$$\alpha\left(\frac{u'}{c'}|_{q^C_*}-1-\eta\right)=\mu \qquad (14)$$

$$\alpha\left(\frac{u'}{c'}|_{q_*^E}-1\right)=\mu. \tag{15}$$

Now as shown in Appendix, (13) can be simplified as

$$\begin{aligned}\bar{\Delta}(E,z_t) &= \left(\frac{1}{\rho+k}\right)\{\alpha[u(q_*^E)-c(q_*^E)]-\mu c(q_*^E)\}- \\ &\left(\frac{1}{\rho+k}\right)\left\{\alpha[u(q_*^C)-c(q_*^C)-\eta c(q_*^C)]-\mu c(q_*^C)\right\}- \\ &\left(\frac{\alpha+\mu\breve{\theta}}{k}\right)\left[\int_{X^*}^{1}\left(\frac{1-X}{1-X^*}\right)^{\frac{\rho}{k}}\omega(z_t,X)dX+\int_0^{X^*}\left(\frac{X}{X^*}\right)^{\frac{\rho}{k}}\omega(z_t,X)dX\right]\end{aligned} \tag{16}$$

where the per-transaction cost $\omega(\cdot)$ is given by (1). Now the cut-off $z^*$ can be defined as $\lim_{k\to\infty}\bar{\Delta}(E,z_t)=0$.

**Proposition 2** *Suppose $\tau\to 0$ and then $k\to\infty$. The division line $Z_\infty$ is horizontal at $z^*$ where $z^*$ is defined by*

$$\begin{aligned}&\{\alpha[u(q_*^E)-c(q_*^E)]-\mu c(q_*^E)\}-\left\{\alpha[u(q_*^C)-c(q_*^C)-\eta c(q_*^C)]-\mu c(q_*^C)\right\} \\ =\ &\left(\alpha+\mu\breve{\theta}\right)\left[\int_0^1\omega(z^*,X)dX\right].\end{aligned} \tag{17}$$

The result above implies that even though the friction on switching means of payment is small enough, the presence of an exogenous shock on the primitive of electronic payment technology can lead to an unique outcome. But $Z_\infty$ is now constant and does not depend on $X$ because when agents are free to switch, observing current payment pattern conveys no information about the future payment pattern.

We now examine the effect of inflation on the willingness to coordination on using E-payment system. Notice that the derivative of the left-hand side with respect to $\mu$ is $[c(q_*^C)-c(q_*^E)]$ which is strictly negative because $q_*^E>q_*^C>0$ from (14) and (15). Meanwhile, the right-hand side of (17) is strictly increasing in $\mu$. Therefore, a higher inflation rate $\mu$ implies a higher threshold $z^*$ because $\omega$ declines as $z$ increases from (1) and $\Omega'(z)<0$. Put differently,

a higher inflation rate has disturbs the coordination on using debit cards due to real balance effect of inflation. That is, $q_*^E > q_*^C$ from (14) and (15), implies $m^E > m^C$. Hence, compared to cash traders, debit-card traders are more susceptible to inflation and the relative benefit of E-payment over C-payment decreases as inflation goes up. It is worthwhile to note that this result is in stark contrast to Monnet and Roberds (2008) where inflation can stimulate the use of credit cards. This difference is mainly attributed to the fact that a credit card is a deferred payment instrument, whereas a debit card is a non-deferred payment one.

Finally, as mentioned, $\theta$ is non-distortionary taxation on buyer's labor, whereas $\breve{\theta}$ is distortionary taxation on buyer's consumption. Then, (17) implies that non-distortionary taxation to finance the E-payment cost encourages the coordination in the use of E-payment: i.e., a higher $\theta$ implies a lower threshold $z^*$. The background channel for this is very similar to that for $\mu$. That is, $q_*^E$ is constant from (15) and then as $\breve{\theta}$ increases, $m^E$ should increase to compensate a seller for bearing more cost of E-payment. Since money holdings are subject to inflation cost if it is not used in the bilateral meeting, a higher $\breve{\theta}$ (a lower $\theta$) renders agents less coordinate in the use of E-payment.

# 5. Concluding Remarks

One of the crucial features of electronic payment technology is that it requires substantial fixed cost but its operation cost is relatively small. Hence, as more people use the technology, per-transaction cost of it declines, economy of scale. This feature implies how agents end up the coordination on using electronic payments would be a critical ingredient in explaining its adoption. However, existing relevant literature does not say much about how to coordinate in the use of electronic payment instrument.

This paper tried to solve this coordination problem in the context of global game reasoning. We show that the presence of exogenous shocks which have a fundament impact on

the cost of the electronic payment technology can resolve an indeterminacy problem about agents beliefs. Furthermore, it turns out that a higher inflation and distortionary financing scheme for the cost of E-payment tend to disturb the coordination on the use of debit cards.

Finally, we might consider various alterations of cost structures about E-payment and C-payment. That is, we here assume that C-payment cost is proportional, whereas E-payment cost is fixed. From the real-world perspective, one might also consider the case where a seller also incurs some cash-handing cost and the case where C-payment cost is also fixed.

# 6. Appendix

**Derivation of (10):** Notice that if a buyer is locked in $o \in \{E, C\}$, his expected payoff with respect to the individual shock can be expressed as

$$W_t^{o_t}(m_t^{o_t}, z_t) = \phi_t(m_t^{o_t} + b_t) + e^{-\rho\tau} \left\{ \begin{array}{c} \alpha\tau[\mathbb{E}_z(V_{t+\tau}^{o_t}(m_{t+\tau}^{o_t}, z_{t+\tau})) - \phi_{t+\tau} m_{t+\tau}^{o_t}] \\ +(\phi_{t+\tau} - e^{\rho\tau}\phi_t) m_{t+\tau}^{o_t} \end{array} \right\}$$
$$+ e^{-\rho\tau}(1 - \alpha\tau) \left\{ \begin{array}{c} k\tau\mathbb{E}_z \max_{o_{t+\tau}} [W_{t+\tau}^{o_{t+\tau}}(0, z_{t+\tau})] \\ +(1 - k\tau)\mathbb{E}_z [W_{t+\tau}^{o_t}(0, z_{t+\tau})] \end{array} \right\}. \tag{18}$$

Now substituting (4) into (18), we can obtain (10):

$$W_t^{o_t}(m_t^{o_t}, z_t) = \phi_t(m_t^{o_t} + b_t) + e^{-\rho\tau} \left\{ \begin{array}{c} \alpha\tau\mathbb{E}_z[u(q_{t+\tau}^{o_t}) - \phi_{t+\tau} p_{t+\tau}^{o_t} - \xi_{t+\tau}^{o_t}] \\ +(\phi_{t+\tau} - e^{\rho\tau}\phi_t) m_{t+\tau}^{o_t} \end{array} \right\}$$
$$+ e^{-\rho\tau} \left\{ \begin{array}{c} k\tau\mathbb{E}_z \max_{o_{t+\tau}} [W_{t+\tau}^{o_{t+\tau}}(0, z_{t+\tau})] \\ +(1 - k\tau)\mathbb{E}_z [W_{t+\tau}^{o_t}(0, z_{t+\tau})] \end{array} \right\}.$$

**Derivation of (11):** The relative payoff to choosing $E$-payment, $W_t^E(m_t^E, z_t) - W_t^C(m_t^C, z_t)$, can be expressed as follows:

$$
\begin{aligned}
W_t^E(m_t^E, z_t) - W_t^C(m_t^C, z_t) = e^{-\rho\tau} & \left\{ \begin{array}{c} \alpha\tau\mathbb{E}_z[u(q_{t+\tau}^E) - \phi_{t+\tau}p_{t+\tau}^E - \theta\omega_{t+\tau}] \\ +(\phi_{t+\tau} - e^{\rho\tau}\phi_t)m_{t+\tau}^E \end{array} \right\} \\
& - e^{-\rho\tau} \left\{ \begin{array}{c} \alpha\tau[u(q_{t+\tau}^C) - \phi_{t+\tau}p_{t+\tau}^C - \eta\phi_{t+\tau}p_{t+\tau}^C] \\ +(\phi_{t+\tau} - e^{\rho\tau}\phi_t)m_{t+\tau}^C \end{array} \right\} \\
& + e^{-\rho\tau}(1-k\tau)\mathbb{E}_z\left[W_{t+\tau}^E(0, z_{t+\tau}) - W_{t+\tau}^C(0, z_{t+\tau})\right].
\end{aligned}
$$

Now substituting successively, the right-hand side can be rearranged as

$$
\sum_{i=0}^{\infty} e^{-\rho\tau i}(1-k\tau)^{i-1}\mathbb{E}_z \left\{ \begin{array}{c} \alpha\tau[u(q_{t+i\tau}^E) - \phi_{t+i\tau}p_{t+i\tau}^E - \theta\omega_{t+i\tau}] \\ +(\phi_{t+i\tau} - e^{\rho\tau}\phi_{(i-1)\tau+t})m_{t+t\tau}^E \end{array} \right\} -
$$

$$
\sum_{i=0}^{\infty} e^{-\rho\tau i}(1-k\tau)^{i-1} \left\{ \begin{array}{c} \alpha\tau[u(q_{t+i\tau}^C) - \phi_{t+i\tau}p_{t+i\tau}^C - \eta\phi_{t+i\tau}p_{t+i\tau}^C] \\ +(\phi_{t+i\tau} - e^{\rho\tau}\phi_{(i-1)\tau+t})m_{t+t\tau}^C \end{array} \right\}.
$$

Since a buyer compensates $\breve{\theta}\mathbb{E}_z\omega_{t+i\tau}$ to a seller, $c(q_{t+i\tau}^E) + \breve{\theta}\mathbb{E}_z\omega_{t+i\tau} = \phi_{t+i\tau}p_{t+i\tau}^E$, we finally have (11):

$$
\begin{aligned}
& W_t^E(m_t^E, z_t) - W_t^C(m_t^C, z_t) = \\
& \qquad \sum_{i=1}^{\infty} e^{-\rho\tau i}(1-k\tau)_z^{i-1}\mathbb{E} \left\{ \begin{array}{c} \alpha\tau u(q_{t+i\tau}^E) + \left(1 - e^{\rho\tau}\frac{\phi_{(i-1)\tau+t}}{\phi_{t+i\tau}} - \alpha\tau\right)c(q_{t+i\tau}^E) \\ + \left[\left(1 - e^{\rho\tau}\frac{\phi_{(i-1)\tau+t}}{\phi_{t+i\tau}}\right)\breve{\theta} - \alpha\tau\right]\omega_{t+i\tau} \end{array} \right\} - \\
& \qquad \sum_{i=1}^{\infty} e^{-\rho\tau i}(1-k\tau)^{i-1} \left\{ \begin{array}{c} \alpha\tau[u(q_{t+i\tau}^C) - \phi_{t+i\tau}p_{t+i\tau}^C - \eta\phi_{t+i\tau}p_{t+i\tau}^C] \\ +(\phi_{t+i\tau} - e^{\rho\tau}\phi_{(i-1)\tau+t})m_{t+t\tau}^C \end{array} \right\}.
\end{aligned}
$$

**Derivation of (12):** The relative payoff to choosing $E$-payment with $\tau \to 0$ and then

$k \to \infty$ can be expressed as follows:

$$\lim_{\tau\to 0}\sum_{i=1}^{\infty} e^{-\rho\tau i}(1-k\tau)_z^{i-1}\mathbb{E}\left\{\begin{array}{c}\alpha\tau u(q^E_{t+i\tau}) + \left(1-e^{\rho\tau t\frac{\phi_{(i-1)\tau+t}}{\phi_{t+i\tau}}}-\alpha\tau\right)c(q^E_{t+i\tau}) \\ +\left[\left(1-e^{\rho\tau t\frac{\phi_{(i-1)\tau+t}}{\phi_{t+i\tau}}}\right)\breve{\theta}-\alpha\tau\right]\omega_{t+i\tau}\end{array}\right\} -$$

$$\lim_{\tau\to 0}\sum_{i=1}^{\infty} e^{-\rho\tau i}(1-k\tau)^{i-1}\left\{\begin{array}{c}\alpha\tau[u(q^C_{t+i\tau})-\phi_{t+i\tau}p^C_{t+i\tau}-\eta\phi_{t+i\tau}p^C_{t+i\tau}] \\ +\left[\phi_{t+i\tau}-e^{\rho\tau}\phi_{(i-1)\tau+t}\right]m^C_{t+t\tau}\end{array}\right\}$$

which can be rearranged as

$$\mathbb{E}_{z_t}\left\{\begin{array}{c}\int_0^\infty e^{-(k+\rho)v}\{\alpha[u(q^E_{t+v})-c(q^E_{t+v})-\omega_{t+v}] \\ -\mu_{t+v}c(q^E_{t+v})-\mu_{t+v}\breve{\theta}\omega_{t+v}\}dv\end{array}\right\}$$
$$-\int_0^\infty e^{-(k+\rho)v}\left\{\alpha[u(q^C_{t+v})-\phi_{t+v}p^C_{t+v}-\eta\phi_{t+v}p^C_{t+v}]-\phi_{t+v}\mu_{t+v}m^C_{t+v}\right\}dv.$$

**Derivation of (16):** Notice that at $Z_\infty$, an agent is indifferent between two means of payment, if she expects all other agents to pick E-payment to the right and C-payment to the left. Since the movement in $X$ always pulls the state away from $Z_\infty$, the dynamics of $(X, z)$ are unstable. With $k \to \infty$, the movement in $X$ is fast relative to the movement in $z$. The system very quickly bifurcates, either upward (all agents use E-payment) or downward (all agents use C-payment). By Theorem 2 and Corollary 1 in Krzysztof, Frankel and Pauzner (1998), as the friction shrinks to zero, the amount of time that passes before a bifurcation occurs goes to zero. Starting with a state $(X^*, z^*)$, the chance of bifurcating up converges to $1 - X^*$, while the chance of bifurcating down goes to $X^*$. Also, the reasoning behind Theorems 1 and Theorem 2 in Burdzy, Frankel and Pauzner (2001) imply that we can ignore what happens in the distant future after $X$ approaches zero or one. In other words, $z$ can be considered constant (relative to the movement of $X$) and is equal to $z^*$. The dynamics of $X$ can be considered approximately as $X^{\uparrow}_{t+v} = 1-(1-X^*)e^{-kv}$ with probability

$(1-X^*)$, and $X^{\downarrow}_{t+v} = X^* e^{-kv}$ with probability $X^*$. And we only need to consider the process of $X$ reaching zero or one. Therefore, if we consider a constant inflation rate $\mu$, the agent's relative payoff to choosing $E$-payment can be approximated as follows:

$$
\begin{aligned}
&(1-X^*)\left\{\int_0^\infty e^{-(k+\rho)v}\{\alpha[u(q_*^E)-c(q_*^E)-\omega^{\uparrow}_{t+v}]-\mu c(q_*^E)-\mu\breve{\theta}\omega^{\uparrow}_{t+v}\}dv\right\}\\
&+X^*\left\{\int_0^\infty e^{-(k+\rho)v}\{\alpha[u(q_*^E)-c(q_*^E)-\omega^{\downarrow}_{t+v}]-\mu c(q_*^E)-\mu\breve{\theta}\omega^{\downarrow}_{t+v}\}dv\right\}\\
&-\int_0^\infty e^{-(k+\rho)v}\{\alpha[u(q_*^C)-c(q_*^C)-\eta c(q_*^C)]-\mu c(q_*^C)\}dv
\end{aligned}
$$

where $\omega^{\uparrow}_{t+v}$ ($\omega^{\downarrow}_{t+v}$) is the path associated with $X^{\uparrow}_{t+v}$ ($X^{\downarrow}_{t+v}$). By rearranging the above, we can finally obtain (16):

$$
\begin{aligned}
&\int_0^\infty e^{-(k+\rho)v}\left\{\alpha[u(q_*^E)-c(q_*^E)]-\mu c(q_*^E)\right\}dv\\
&-\int_0^\infty e^{-(k+\rho)v}\left\{\alpha[u(q_*^C)-c(q_*^C)-\eta c(q_*^C)]-\mu c(q_*^C)\right\}dv\\
&-(1-X^*)\left\{\int_0^\infty e^{-(k+\rho)v}(\mu\breve{\theta}+\alpha)\omega^{\uparrow}_{t+v}dv\right\}-X^*\left\{\int_0^\infty e^{-(k+\rho)v}(\mu\breve{\theta}+\alpha)\omega^{\downarrow}_{t+v}dv\right\}\\
&=\left(\frac{1}{\rho+k}\right)\left\{\alpha[u(q_*^E)-c(q_*^E)]-\mu c(q_*^E)-\alpha[u(q_*^C)-c(q_*^C)-\eta c(q_*^C)]+\mu c(q_*^C)\right\}\\
&-\left(\frac{\alpha+\mu\breve{\theta}}{k}\right)\left[\int_{X^*}^1\left(\frac{1-X}{1-X^*}\right)^{\frac{\rho}{k}}\omega(z_t,X)dX+\int_0^{X^*}\left(\frac{X}{X^*}\right)^{\frac{\rho}{k}}\omega(z_t,X)dX\right].
\end{aligned}
$$

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