

# Optimal Allocation of Social Cost for Electronic Payment System: A Ramsey Approach

PIDONG HUANG<sup>\*</sup>, YOUNG SIK KIM<sup>†</sup>, MANJONG LEE<sup>‡</sup>

## Abstract

Using a standard Ramsey approach, we examine an optimal allocation of the social cost for electronic payment system in the context of a dynamic general equilibrium model where money is essential. The benevolent government provides electronic payment services and allocates the relevant social cost through taxation on the beneficiaries' labor and consumption. A higher tax rate on labor yields the following desirable allocations. First, it implies a lower welfare loss due to the distortionary consumption taxation. It also enhances the economy of scale in the use of an electronic payment technology, reducing per transaction cost of electronic payment. Finally, it saves the cost of withdrawing and carrying around cash by reducing the frequency of cash trades. All these channels together imply an optimality of the zero tax rate on consumption.

*Keywords:* cash, electronic payment, social cost, Ramsey problem

*JEL classification:* E40, E41

---

<sup>\*</sup>Department of Economics, Korea University, Email: pidonghuang@korea.ac.kr

<sup>†</sup>Department of Economics, Seoul National University, Email: kimy@snu.ac.kr

<sup>‡</sup>Department of Economics, Korea University, Email:manjong@korea.ac.kr

# 1. Introduction

Electronic payment system requires record-keeping and information-process technologies such as clearing and verification systems. In the presence of cash as a readily available means of payment, activation of such technologies incurs additional resource costs from the society's point of view. Hayashi and Keeton (2012) call this resource costs as "social cost" of using electronic payment methods and distinguish it from "private cost" which includes the fees imposed by one party to another. This social cost has the distinct feature of being largely composed of fixed cost which does not rely on the frequency or the value of transactions. That is, once the electronic record-keeping and information-process devices are in place at a substantial cost, electronic payment services could be provided with trivial marginal cost per transaction.

Considering such a high social cost incurred in the installment of electronic payment technology, it is quite natural to ask how the social cost should be allocated. We explore this question by incorporating a standard Ramsey approach into a dynamic general equilibrium model where money is essential.

More specifically, we adopt a search theoretic monetary model of Shi (1995) and Trejos and Wright (1995) augmented with distribution of wealth. At the beginning of a period, each agent is randomly matched with another agent. In pairwise meetings, agents cannot commit to their future actions and trading histories are private, which rules out any possibility of credit trades. Hence, all trades in a pairwise meeting are *quid pro quo* and either cash or checking-account deposit should be transferred in exchange for goods produced where the terms of trade are determined by a buyer's take-it-or-leave-it offer.

A buyer who is willing to pay with cash can withdraw it from her checking account at some cost, whereas users of electronic payment technology bear its fixed activating cost. That is, electronic transactions require the installment of an electronic payment system which incurs

the government (service provider) a fixed cost. The government collects it from people who transfer and receive money via the system. Therefore, per transaction cost of electronic payment declines as more transactions are made via the electronic payment system. This captures an economy of scale in the use of electronic payment technology. As a key source of efficiency for electronic payment, this economy of scale relies on the choice of means of payment by heterogenous agents with different monetary wealth.

We used the model to examine an optimal allocation of the social cost for electronic payment system. Our Ramsey problem is very standard—that is, in order to maximize social welfare, the government chooses a policy on how to allocate the social cost for electronic payment system across buyers and sellers who use the system in pairwise trades. Under the assumed buyers-take-all trading protocol, this problem is transformed into that of choosing a tax scheme on the beneficiaries (i.e., buyers) of electronic transactions in the form of taxation on labor or on consumption.

The choice of taxation on a buyer’s labor or consumption has not only an intensive-margin effect on the terms of trade in pairwise meetings but also an extensive-margin effect on the choice of means of payment. In particular, the labor taxation has a lump-sum feature in the sense that it does not affect quantity consumed in exchange for money transferred electronically, whereas the consumption taxation is distortionary in the sense that it affects quantity consumed in the electronic transactions.

Notwithstanding a lump-sum feature of the labor taxation, it is not obvious at all whether a higher tax rate on labor improves welfare. First of all, for a given cost per electronic transaction, labor taxation is preferred for relatively poor agents whose consumption is small enough, whereas consumption taxation is preferred for relatively rich agents whose consumption is sufficiently large. What is more, the tax rate on labor has an extensive-margin effect on the choice of means of payment which then gives an effect on the cost per electronic transaction via an economy-of-scale channel. These imply that an optimal allocation

of the social cost for electronic payment system would depend crucially on nondegenerate distribution of wealth.

Since the endogeneity of nondegenerate wealth distribution rules out close-form solutions, the Ramsey problem is solved numerically. Our results show that the frequency of electronic transactions increase as the tax rate on labor increases. With the zero tax rate on consumption good, for instance, a buyer prefers an electronic payment method to cash if the cash-withdrawing cost exceeds the tax burden associated with using an electronic payment method. Even in such a case, however, a buyer with sufficiently high marginal utility of consumption is willing to pay in cash with the zero tax rate on labor. Therefore, the government policy that increases the tax rate on labor enhances the economy of scale, which then decreases per transaction cost of electronic payment further. This eventually amplifies an extensive-margin effect on the use of electronic payment method.

In sum, welfare loss due to taxation on consumption, cash-withdrawing cost, and per transaction cost of electronic transaction decreases with the tax rate on labor. As a result, the zero tax rate on consumption good is an optimal method to allocate the social cost for electronic payment technology.

The paper is organized as follows. Section 2 describes the model economy. Section 3 defines a symmetric stationary equilibrium. Section 4 formulates the Ramsey problem for the benevolent government and explores an optimal allocation of the social cost for electronic payment system. Section 5 summarizes the paper with a few concluding remarks.

## 2. Model

The background environment is a standard random matching model of money in the tradition of Shi (1995) and Trejos and Wright (1995) augmented with distribution of money holdings.<sup>1</sup>

---

<sup>1</sup>Among the other random matching models that consider the distribution of money holdings are Green and Zhou (1998, 2002), Camera and Corbae (1999), Zhu (2003), Berentsen, Camera and Waller (2005), and

Time is discrete. There is a  $[0, 1]$  continuum of each of  $N \geq 3$  types of infinitely lived agents with  $N$  distinct types of divisible and perishable goods produced at each period. A type  $n \in \{1, 2, \dots, N\}$  agent produces only good  $n$  which incurs a disutility cost of  $c(q) = q$  for producing  $q$  units of output, whereas a type- $n$  agent consumes only good  $n + 1$  (modulo  $N$ ) which gives an utility of  $u(q)$  for consuming  $q$  units of output where  $u'' < 0 < u'$ ,  $u(0) = 0$ ,  $u'(0) = \infty$ , and  $u'(\infty) = 0$ . Each agent maximizes expected discounted utility with a discount factor  $\beta \in (0, 1)$ .

There are three exogenous nominal quantities that describe the stock of money: upper bound on individual money holdings, size of the smallest unit of money, and average monetary holdings per each specialization type. We normalize the smallest unit to be unity so that the set of possible individual money holdings consists of integer numbers, namely,  $\mathbb{M} = \{0, 1, 2, \dots, M\}$  where  $M > 0$  denotes the upper bound required for compactness. We denote average money holdings per each specialization type by  $\bar{m} > 0$ .

Each agent enters a period with some amount of monetary wealth and can freely deposit it into a checking account. The government has an intra-temporal record-keeping technology on checking accounts, but not on agents' trading histories. Other than the account-related tasks, the government does not engage in any other economic activities, including consumption or production of any goods.

In each period, an agent is randomly matched with another agent. Trades can occur only in single-coincidence meetings between type- $n$  and type- $(n + 1)$  agents. Agents in a meeting know each other's specialization type and money holdings. However, trading histories are private and people cannot commit to future actions, which rules out any possibility of credit trades and makes a medium of exchange essential (see, for instance, Kocherlakota 1998, Wallace 2001, Corbae, Temzelides and Wright 2003, and Aliprantis, Camera and Puzello 2007).

---

Molico (2006).

In a single-coincidence meeting, a buyer makes take-it-or-leave-it offer  $(q, p)$  to a seller where  $q$  denotes quantity of goods produced by the seller for the buyer and  $p$  denotes the amount of money transferred by the buyer to the seller. Transactions via an electronic payment method require an electronic payment system to be activated, which incurs the government (service provider) a fixed cost of  $\Omega$  in installing the system. In practice, the setting up of electronic payment system entails substantial fixed cost, but trivial marginal cost per transaction. In order to focus on the former, the latter is assumed to be zero. Hereinafter a debit card will be regarded as the representative electronic payment method in the sense that it is one of the primary electronic means of payment for in-store purchases and typically lacks in credit function.

In order to provide debit-card service to the society, the government should raise resources  $\Omega$  required to pay for the social cost of electronic payment system. Noting that the government can access within-period checking-account information only, taxation on no-traders or cash traders is not feasible because their transactions are not in the government information system. On the other hand, checking accounts should be cleared between a payer and a payee through the electronic payment system operated by the government. Hence, debit-card transactions are effectively notified to the government, so that the government can collect the required resources from debit-card traders. Specifically, the government raises the social cost  $\Omega$  by levying  $\omega$  per debit-card transaction where  $\omega$  satisfies the following government's budget constraint:

$$\Omega = \mathbf{S} (\tau\omega + \tau^c\omega). \quad (1)$$

Here  $\mathbf{S}$  denotes the frequency of debit-card transactions,  $\tau \in [0, 1]$  is the share of social cost allocated to a buyer, and  $\tau^c = (1 - \tau) \in [0, 1]$  is that to a seller. For a given  $(\Omega, \tau)$ ,  $\mathbf{S}$  captures an economy of scale in the use of electronic payment technology in the sense that a higher  $\mathbf{S}$  implies a lower  $\omega$ . We assume that debit-card traders pay the cost to the government by

producing their specialized types of goods. With the bargaining rule of a buyer's take-it-or-leave-it offer, the cost share  $\tau \in [0, 1]$  allocated to a buyer can be interpreted as a taxation on the buyer's labor required to produce output for the establishment of electronic payment system. On the other hand, the cost share  $\tau^c \in [0, 1]$  allocated to a seller can be interpreted as a taxation on the buyer's consumption required to set up electronic payment system. (For more concrete exposition, see Section 4.)

If a buyer uses a debit card to transfer  $p$  amount of money to a seller, it is deducted from the buyer's account and deposited into the seller's account immediately. A buyer can also pay  $p$  in cash by carrying it from the beginning of a period at the disutility cost of  $\tilde{\eta}p$  or by withdrawing it from her account at the disutility cost of  $\eta p$ . Since the cash-carrying cost  $\tilde{\eta}$  includes foregone interest (e.g., Li 2011) as well as the inconvenience and the risk of loss (or theft) associated with carrying cash around, we assume  $\tilde{\eta} \geq \eta$ . Now, if an agent carries  $p$  amount of cash into a pairwise meeting, she can save the cash withdrawing cost ( $\eta p$ ) with probability  $(1/N)$  but will bear the cost  $\tilde{\eta}p$  which is greater than  $(\eta p/N)$ . This implies that at the beginning of a period, all agents deposit their money into checking accounts, which simplifies our exposition considerably by rendering a straightforward deposit-decision problem.<sup>2</sup>

For a type- $n$  agent, per-period utility is given by

$$u(q_{n+1}) - \tilde{q}_n - \tau\omega\mathbb{I} - \eta c\check{\mathbb{I}}. \quad (2)$$

where  $q_{n+1}$  is consumption of good  $n + 1$ ,  $\tilde{q}_n$  is production of good  $n$  (including the share of social cost  $\tau^c\omega$  by accepting debit cards),  $\mathbb{I}$  is an indicator function that equals 1 if a type- $n$  agent meets a type- $(n + 1)$  agent and make a trade via a debit card,  $\check{\mathbb{I}} = 1 - \mathbb{I}$ , and  $c$  is the

---

<sup>2</sup>In Section 4, we consider the case in which proportional cost  $\eta$  is replaced with fixed cost. Also we here assume that a cash trade does not incur any cost to a seller but in Section 4, we consider the case in which cash-handling cost is borne by a seller.

amount of cash withdrawn by a type- $n$  agent for a bilateral trade.

Finally, at the end of each period, the government redeems the balance in checking account to each agent and then information on each account is wiped out completely. Agents go on to the next period with the end-of-period money holdings.

### 3. Equilibrium

We study a symmetric (across specialization types) and stationary equilibrium. For a given  $(\Omega, \eta)$  and a government policy on  $\tau$ , a symmetric steady state consists of functions  $(v, \pi)$  and  $\omega$  that satisfy the conditions described below. The functions  $v : \mathbb{M} \rightarrow R$  and  $\pi : \mathbb{M} \rightarrow [0, 1]$  pertain to the beginning of a period and prior to the pairwise trades such that  $v(m)$  is the expected discounted value of having monetary wealth  $m$  and  $\pi(m)$  is the fraction of each specialization type with  $m$ .

Consider a generic single-coincidence meeting between a buyer with  $b \in \mathbb{M}$  and a seller with  $s \in \mathbb{M}$ . We let

$$\Gamma(b, s) = \{p : p \in \{0, 1, \dots, \min\{b, M - s\}\}\}. \quad (3)$$

That is,  $\Gamma(b, s)$  is the set of feasible wealth transfers from the buyer to the seller. Noting that all agents deposit their money into checking accounts at the beginning of a period and a seller accepts all offers that leave her no worse off (tie-breaking rule), the buyer's problem can be expressed as

$$\max_{\mathbb{I} \in \{0,1\}} \left\{ \begin{array}{l} \mathbb{I} \{u[\beta v(s + p_d(b, s, v)) - \beta v(s) - \tau^c \omega] + \beta v[b - p_d(b, s, v)] - \tau \omega\} + \\ (1 - \mathbb{I}) \{u[\beta v(s + p_c(b, s, v)) - \beta v(s)] + \beta v[b - p_c(b, s, v)] - \eta p_c(b, s, v)\} \end{array} \right\} \quad (4)$$

where  $p_i(b, s, v)$  for  $i \in \{d, c\}$  denotes respectively

$$p_d(b, s, v) = \arg \max_{p \in \Gamma(b, s)} u [\beta v(s + p) - \beta v(s) - \tau^c \omega] + \beta v(b - p) - \tau \omega \quad (5)$$

$$p_c(b, s, v) = \arg \max_{p \in \Gamma(b, s)} u [\beta v(s + p) - \beta v(s)] + \beta v(b - p) - \eta p. \quad (6)$$

Let  $g(b, s, v)$  be the maximized value of (4) and  $p^*(b, s, v)$  be an associated  $p_i(b, s, v)$ : that is,  $p^*(b, s, v) = p_d(b, s, v)$  if  $\mathbb{I} = 1$  and  $p^*(b, s, v) = p_c(b, s, v)$  if  $\mathbb{I} = 0$ .<sup>3</sup> Since  $p_i(b, s, v)$  for  $i \in \{c, d\}$  is discrete,  $p^*(b, s, v)$  can be multi-valued, in which case we allow for all possible randomizations over them. Let  $\Delta(b, s, v)$  be the set of measures that represents those randomizations. Then  $\Delta(b, s, v)$  can be described as

$$\Delta(b, s, v) = \{\delta(\cdot; b, s, v) : \delta(m; b, s, v) = 0 \text{ if } m \notin \{b - p^*(b, s, v)\}\} \quad (7)$$

where  $\delta(m; b, s, v)$  is the probability that in a single-coincidence meeting the buyer with  $b \in \mathbb{M}$  offers  $b - m$  to the seller with  $s \in \mathbb{M}$ , ending up with  $m$ .

Now we can describe the evolution of wealth distribution induced by pairwise trades as follows:

$$\Pi(v) = \left\{ \pi : \pi(m) = \frac{1}{N} \sum_{(b, s)} \pi(b) \pi(s) [\delta(m) + \delta(b - m + s)] + \frac{N - 2}{N} \pi(m) \text{ for } \delta \in \Delta(b, s, v) \right\}. \quad (8)$$

The first probability measure in the right-hand side of (8) corresponds to single-coincidence meetings, whereas the second corresponds to all other cases. Noting that in (7)  $\delta$  is defined over the post-trade money holdings of the buyer, the buyer's post-trade money holdings ( $b - m + s$ ) corresponds to the seller's post-trade wealth  $m$  ( $= b - (b - m + s) + s$ ). Notice also that the dependence of  $\Pi$  on  $v$  comes from the dependence of  $\delta$  on  $v$  in (7).

Finally, since the payoff as a seller with  $m \in \mathbb{M}$  is simply  $\beta v(m)$ , the expected value of

---

<sup>3</sup>Without loss of generality, we can disregard the case in which some fraction of  $p$  is paid in cash and the remains of  $p$  is paid in a debit card because once a debit card is used,  $\omega$  is imposed regardless of the value of debit-card transaction.

holding  $m$  before pairwise meeting,  $v(m)$ , can be written as

$$v(m) = \frac{1}{N} \sum_s \pi(s)g(m, s, v) + \frac{N-1}{N}\beta v(m). \quad (9)$$

**Definition 1** For given  $(\Omega, \eta, \tau)$ , a symmetric stationary equilibrium is a set of functions  $(v, \pi)$  and  $\omega$  such that (i) the value function  $v$  satisfies (9); (ii) the probability measure  $\pi$  of wealth distribution satisfies  $\pi \in \Pi(v)$  where  $\Pi(v)$  is given by (8); (iii)  $\omega$  satisfies the government's resource constraint,  $\Omega = \omega \sum_{(b,s)} \pi(b)\pi(s)\mathbb{I}(b, s, v)$ .

The existence of a symmetric stationary equilibrium for some parameters is a straightforward extension of the existence results in Zhu (2003) and Lee, Wallace and Zhu (2005): If  $(\bar{m}, M/\bar{m})$  are large enough respectively, and  $\Omega$  and  $\eta$  are not too large respectively, then there exists a monetary symmetric stationary equilibrium  $(v, \pi, \omega)$  with  $v$  strictly increasing and strictly concave.

## 4. Optimal Allocation of Social Cost

We now examine an optimal allocation of social cost for electronic payment system using a standard Ramsey taxation approach.

### 4.1. The Ramsey Problem

In order to formulate a Ramsey problem for the benevolent government as a provider of electronic payment services, we first define welfare as the lifetime expected discounted utility of a representative agent before the assignment of wealth according to a stationary distribution.

Let  $\mathbb{W}_\tau$  denote the welfare of a stationary equilibrium  $(v_\tau, \pi_\tau, \omega_\tau)$  for a given policy  $\tau$ . Then  $\mathbb{W}_\tau$  can be expressed as follows:

$$\mathbb{W}_\tau = \frac{\pi_\tau \mathbf{U} \pi'_\tau}{(1-\beta)N}. \quad (10)$$

Here the element in row  $b \in \mathbb{M}$  and column  $s \in \mathbb{M}$  of the matrix  $\mathbf{U}$  is

$$u[q(b, s, v_\tau)] - \tilde{q}(b, s, v_\tau) - \tau\omega\mathbb{I}(b, s, v_\tau) - \eta c(b, s, v_\tau)\check{\mathbb{I}}(b, s, v_\tau)$$

with  $c(b, s, v_\tau)$  denoting the amount of cash withdrawn for the single-coincidence meeting and  $\tilde{q}(b, s, v_\tau) = q(b, s, v_\tau) + \tau^c\omega\mathbb{I}(b, s, v_\tau)$  where the second term,  $\tau^c\omega\mathbb{I}(b, s, v_\tau)$ , captures the disutility borne by the seller from accepting a debit card.

Under the buyer's take-it-or-leave-it trading protocol, the benefit principle is implementable in the sense that the social cost for electronic payment system is borne by its beneficiaries (i.e., buyers) regardless of  $\tau$ . That is, from (4), in a pairwise meeting between a buyer with  $b \in \mathbb{M}$  and a seller with  $s \in \mathbb{M}$ , the buyer's net payoff for  $\mathbb{I} = 1$  is

$$u[q(b, s, v_\tau) - \tau^c\omega] - \tau\omega + \beta\{v_\tau[b - p_d(b, s, v_\tau)] - v_\tau(b)\}$$

where  $q(b, s, v_\tau) = \beta v_\tau(s + p_d) - \beta v_\tau(s)$  and  $\tau + \tau^c = 1$ . Therefore, the choice of  $\tau$  can be interpreted as allocating the social cost to the beneficiaries of electronic transactions in the form of labor taxation ( $\tau = 1, \tau^c = 0$ ) or consumption taxation ( $\tau = 0, \tau^c = 1$ ) or a certain combination of the two taxations [ $\tau, \tau^c \in (0, 1)$ ]. It is worth noting in the above equation that the labor taxation ( $\tau = 1, \tau^c = 0$ ) has a lump-sum feature in the sense that it does not affect quantity consumed in exchange for money transferred electronically. On the other hand, the consumption taxation ( $\tau = 0, \tau^c = 1$ ) is distortionary in the sense that it affects quantity consumed with the electronic transactions.

Now the Ramsey problem for the government is to choose the tax rate on labor ( $\tau$ ) and the implied tax rate on consumption ( $\tau^c = 1 - \tau$ ) to maximize welfare ( $\mathbb{W}$ ) taking into account its effect on the equilibrium reactions of buyers and sellers in pairwise meetings.

**Definition 2** *The Ramsey problem for the benevolent government is to choose a symmetric*

stationary equilibrium  $(v_\tau, \pi_\tau, \omega_\tau)$  in Definition 1 which maximizes (10), or equivalently to choose  $\tau^* = \arg \max_{\tau \in [0,1]} \mathbb{W}_\tau$ .

In order to quantify the magnitude of welfare loss across different policies, we calculate the welfare cost of  $\tau$ -policy relative to that of an infeasible lump-sum taxation.<sup>4</sup> More specifically, we first find  $\Delta$  that solves

$$\bar{\mathbb{W}} = \frac{\pi_\tau \mathbf{U}_\Delta \pi'_\tau}{(1 - \beta)N} \quad (11)$$

where  $\bar{\mathbb{W}}$  is the welfare with lump-sum taxation. Also, the element in row  $b \in \mathbb{M}$  and column  $s \in \mathbb{M}$  of  $\mathbf{U}_\Delta$  is

$$u[q(b, s, v_\tau) + \Delta] - \tilde{q}(b, s, v_\tau) - \lambda \omega \mathbb{I}(b, s, v_\tau) - \eta c(b, s, v_\tau) \check{\mathbb{I}}(b, s, v_\tau).$$

That is,  $\Delta$  is an additive consumption compensation that makes the welfare with  $\tau$ -policy equal to that with lump-sum taxation ( $\bar{\mathbb{W}}$ ).<sup>5</sup> The welfare cost of  $\tau$ -policy is then calculated as a ratio of  $\Delta$  to the average consumption in the symmetric stationary equilibrium with  $\tau$ -policy.

Even though the labor taxation ( $\tau$ ) has a lump-sum feature, it is not obvious at all whether a higher  $\tau$  improves or deteriorates welfare. First of all, for a given  $\omega$ , a higher  $\tau$  increases an intensive margin (output per unit of money) but its effect on welfare is ambiguous. The marginal gain from an enhanced intensive margin is  $(\partial u / \partial \tau) = u'(q)\omega$ , whereas marginal labor cost from increasing  $\tau$  is just  $\omega$ . Therefore, a higher  $\tau$  is beneficial to buyers whose consumption is small enough so that  $u'(q) > 1$ , whereas it is detrimental

---

<sup>4</sup>If the technological cost is raised by an equal per capita lump-sum taxation, all trades are made by debit cards and trading behaviors are not affected at all. However, as discussed in Section 2, the government cannot force it effectively to no-traders or cash traders.

<sup>5</sup>Consumption compensation is calculated as an addition rather than as a multiple of consumption because consumption is zero in some single-coincidence meetings.

to buyers whose consumption is large enough so that  $u'(q) < 1$ . Also,  $\tau$  has an extensive-margin effect on the choice of means of payment and hence on the economy of scale. That is,  $\omega = (\Omega/\mathbf{S})$  relies on  $\tau$  through the dependence of  $\mathbf{S}$  (frequency of electronic transactions) on  $\tau$ .

The above discussion implies that the effect of  $\tau$  on welfare depends crucially on non-degenerate distribution of wealth across agents which is determined endogenously. This then rules out closed-form solutions and hence, in what follows, we tackle our question based on observations of numerical exercises.

## 4.2. Parameterization

In order to solve the model numerically, we parameterize the basic environment as follows. We first assume  $N = 3$ , the smallest number of types of agents which is consistent with no double-coincidence meeting, and  $\beta = 0.99$ . We set  $(\bar{m}, M) = (40, 3\bar{m})$  so that the indivisibility of money and the upper bound on money holdings are not too severe. In this type of model, almost all monetary offers are either 0 or 1 if the indivisibility of money is too severe. However, it is not the case in our examples below. In addition,  $M = 3\bar{m}$  is large enough in the sense that almost no one is at the upper bound in a stationary equilibrium and hence the result would be hardly affected even if a larger  $M$  were assumed.

We let  $u(q) = \theta \ln(1 + q)$  where  $\theta$  together with  $(\Omega, \eta)$  is chosen to fit the model to the data.<sup>6</sup> Based on the 2011 Survey of Consumer Payment Choice, Shy (2012) reports that  $\mathbf{S} = 0.511$  where  $\mathbf{S}$  is the fraction of debit-card transactions out of cash and debit cards transactions. In addition, according to the Boston Fed's 2010 Diary of Consumer Payment Choice, Stavins (2012) reports that  $\mathbf{D} = 1.65$  where  $\mathbf{D}$  denotes the average value of debit-card transactions to that of cash. In our model,  $(\theta, \Omega, \eta) = (2.1, 2.0 \times 10^{-3}, 8.42 \times 10^{-4})$

---

<sup>6</sup>We do not need to set the value of  $\tilde{\eta}$  because the role of  $\tilde{\eta}$  is just to ensure that all agents deposit their money into checking accounts at the beginning of a period, which is fulfilled for any  $\tilde{\eta}$  such that  $\tilde{\eta} \geq \eta$ .

with  $\tau = 0$  generate  $\mathbf{S} = 0.512$  and  $\mathbf{D} = 1.50$ . We here consider  $\tau = 0$  because in the U.S. economy, almost all the cost of debit-card transaction is imposed to sellers on the surface. Notice that  $\Omega = 2.0 \times 10^{-3}$  corresponds to 0.18% of  $q^* = \arg \max[u(q) - q] = 1.1$  which is very close to the estimate in Aiyagari, Braun and Eckstein (1998).<sup>7</sup>

### 4.3. Labor Taxation vs Consumption Taxation

Figure 1 shows welfare and welfare cost as a function of the tax rate on labor ( $\tau$ ) where the welfare cost for each  $\tau$ -policy is calculated relative to that for an infeasible lump-sum taxation. As shown in Figure 1, welfare increases with the tax rate on labor and the solution to the Ramsey problem in Definition 2 turns out to be  $\tau^* = 1$ . That is, the zero tax rate on consumption good attains the highest welfare. The welfare cost with the zero tax rate on consumption ( $\tau = 1$ ) remains 0.02% only, but it increases up to 0.11% with the zero tax rate on labor ( $\tau = 0$ ).

The underlying mechanism that renders the zero tax rate on consumption optimal is as follows. First, a higher tax rate on labor implies a relatively lower tax rate on consumption and hence consumption distortion is reduced. As shown in Figure 2, the gap between the quantity of good consumed by buyers and that produced by sellers shrinks as the tax rate on consumption decreases. As a consequence, the welfare loss due to the distortionary consumption taxation decreases and eventually converges to zero as the tax rate on labor approaches to 1.

Second, a higher tax rate on labor enhances the economy of scale in the use of electronic payment system. As shown in Figure 3, the frequency of debit-card transactions ( $\mathbf{S}$ ) increases with  $\tau$ , which eventually lessens per debit-card transaction cost ( $\omega = \Omega/\mathbf{S}$ ). The two extreme policies,  $\tau = 1$  and  $\tau = 0$  for a given  $(\omega, \eta)$ , are useful to understand why the frequency of

---

<sup>7</sup>Aiyagari, Braun and Eckstein (1998) estimate that the cost incurred by the U.S. banks in providing demand-deposit services is around 0.2% of GDP.

Figure 1: Welfare and welfare cost as a function of  $\tau$

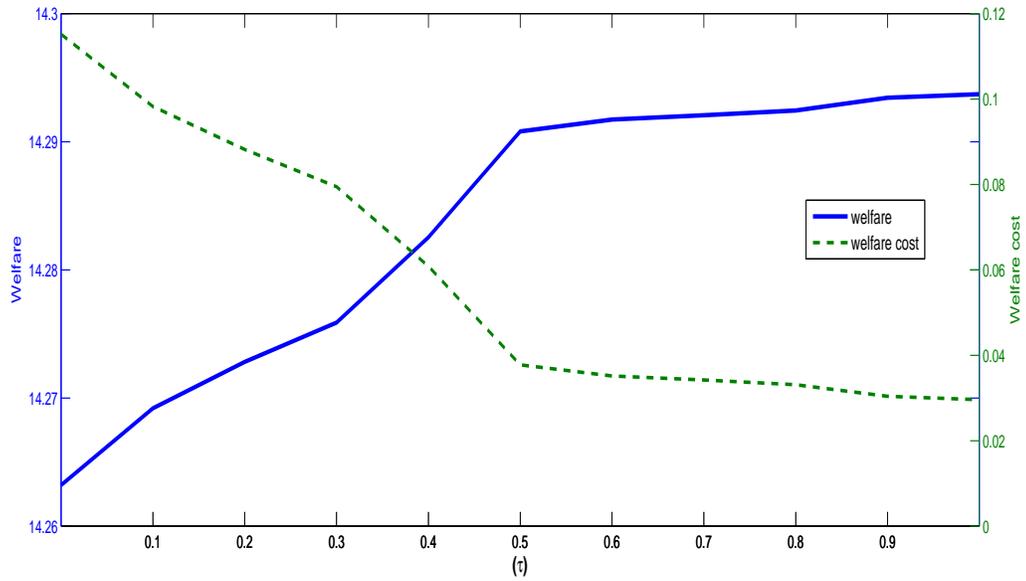
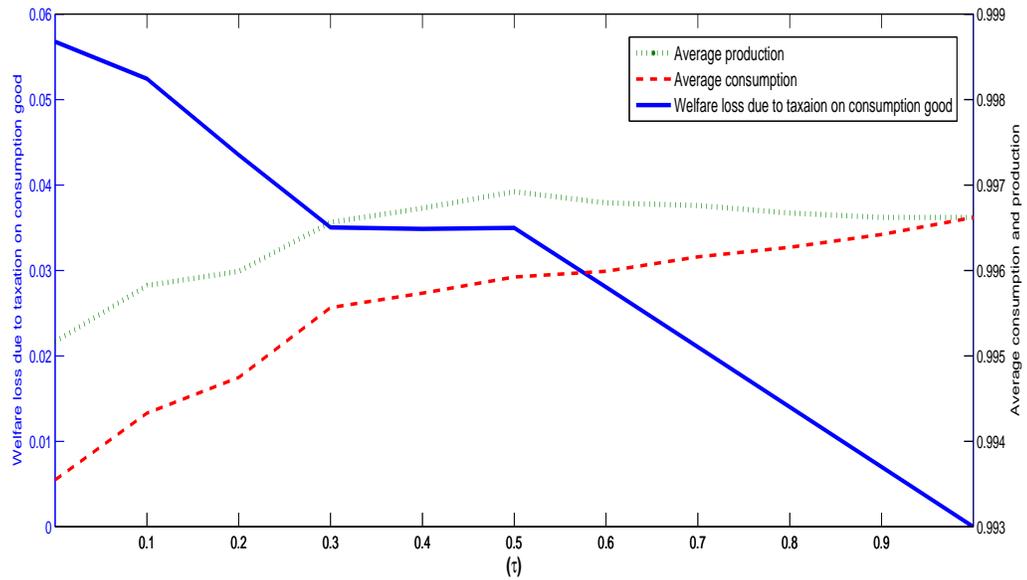
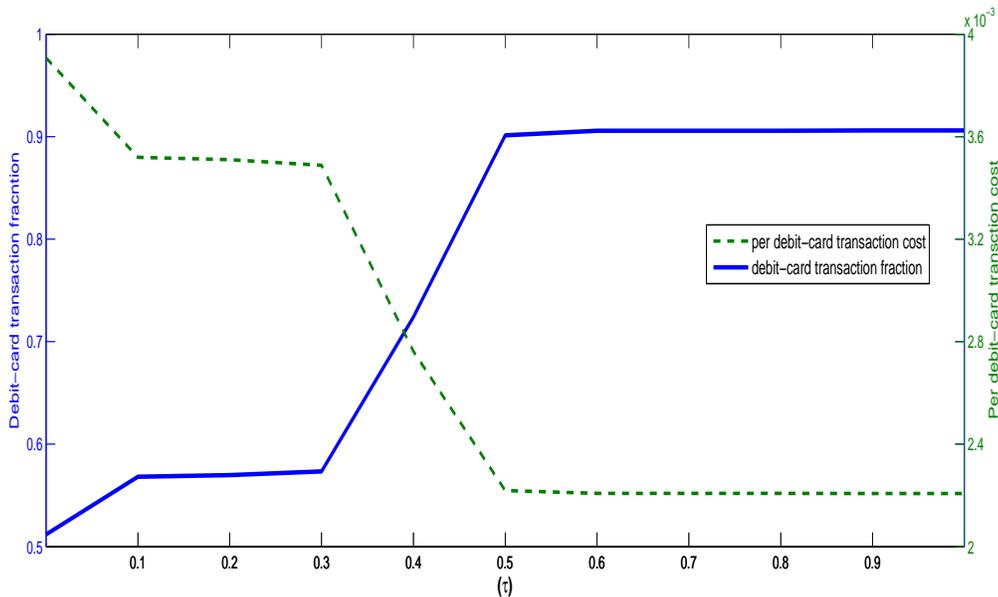


Figure 2: Welfare loss due to taxation on consumption good



electronic transactions ( $\mathbf{S}$ ) increases with  $\tau$ . In the case of  $\tau = 1$ , all transactions will be made by debit cards as long as  $p > (\omega/\eta)$ . However, in the case of  $\tau = 0$ , transactions can be made in cash even if  $p > (\omega/\eta)$ . That is,  $p$  exceeding  $(\omega/\eta)$  will be paid in cash rather than a debit card as long as  $u\{\beta v[s + p(\cdot)] - \beta v(s)\} - u\{\beta v[s + p(\cdot)] - \beta v(s) - \omega\} > \eta p(\cdot)$ . The latter happens in a pairwise meeting when  $\beta v[s + p(\cdot)] - \beta v(s) = q_c$  is sufficiently small. This economy-of-scale enhancement with a higher  $\tau$  for a given  $\omega$  subsequently has the effect of increasing the use of debit cards further by lowering  $\omega$ . Notice that as the extensive margin is enhanced, as shown in Figure 4, the fraction of buyers preferring consumption-good taxation [ $u'(q) < 1$ ] declines.

Figure 3: Frequency of debit-card transactions and cost per transaction



Third, Figure 5 shows that the total cost of withdrawing cash decreases with the tax rate on labor. This is because the frequency of cash transaction decreases as  $\tau$  increases. More transactions are subject to labor taxation and hence the social cost for electronic payment system allocated in the form of labor tax also increases with the tax rate on labor.

Figure 4: Fraction of buyers with  $u'(q) < 1$

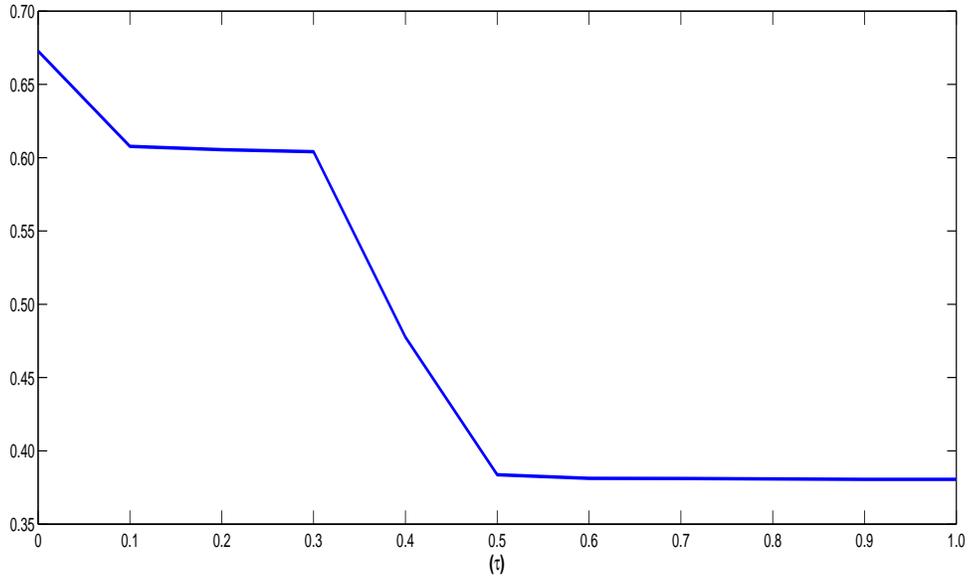
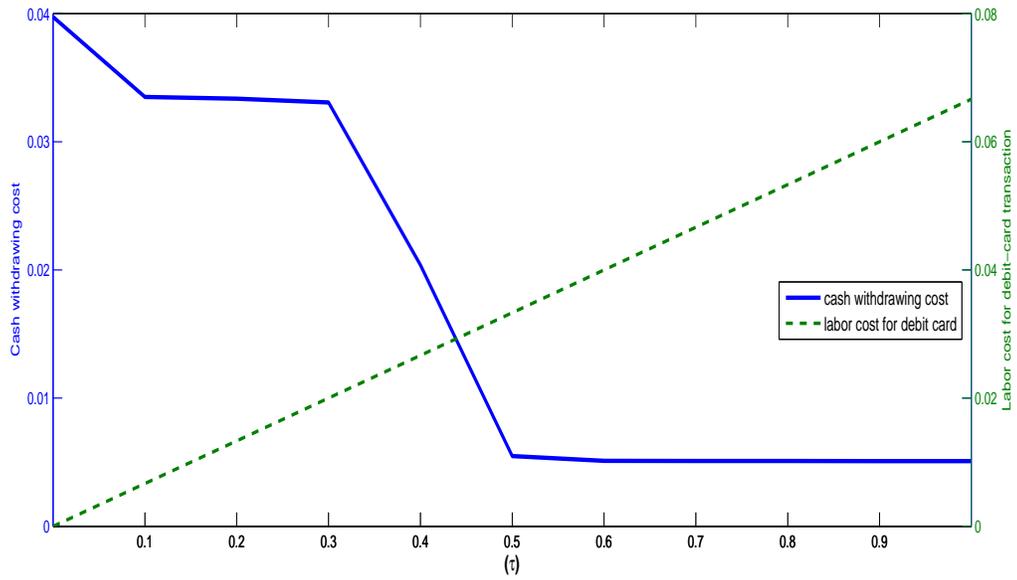
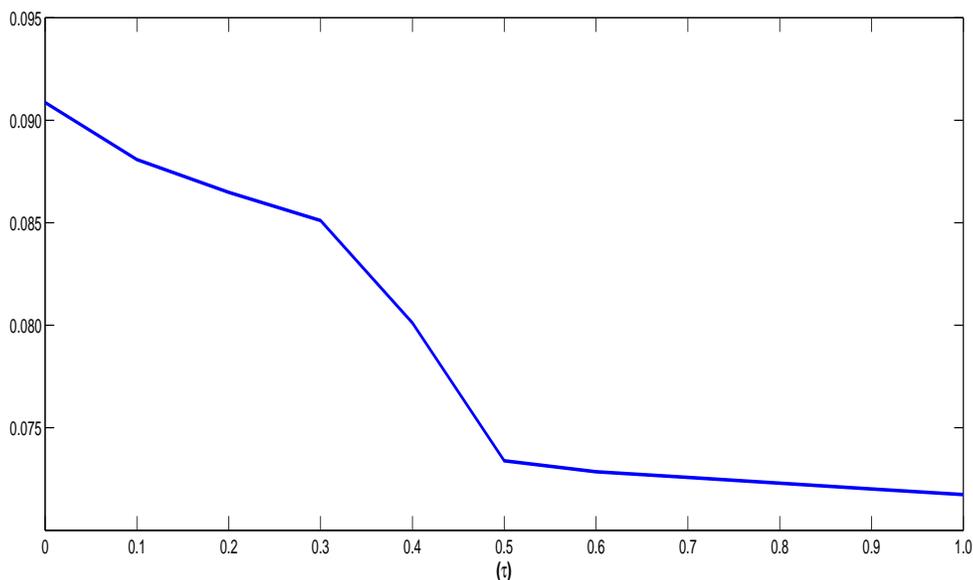


Figure 5: Cash withdrawing cost and labor cost for debit-card transactions



In sum, Figure 6 shows the sum of all costs discussed above such as welfare loss due to the distortionary consumption taxation, cash withdrawing cost and labor resources cost for debit-card transactions as a function of  $\tau$ . An overall cost of transactions decreases with  $\tau$  which immediately implies the results in Figure 1.

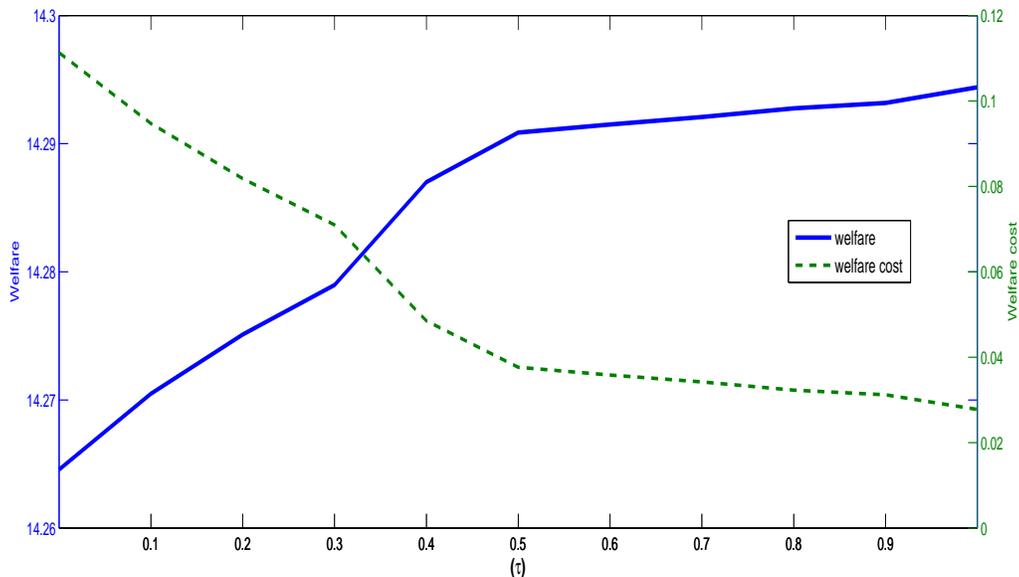
Figure 6: Overall transaction cost as a function of  $\tau$



Finally, we check the robustness of an optimality of zero-tax-rate on consumption good to different settings. We first introduce the fixed cash-handling cost for sellers. That is, a seller accepting cash is borne by a fixed cash-handling cost  $\kappa$ . In computing a steady state for this case, we set  $\kappa$  to fit the model to the U.S. data concerning the fraction of debit-card transaction ( $\mathbf{S} = 0.511$ ) and the ratio of average value of debit-card transactions to that of cash ( $\mathbf{D} = 1.65$ ). The model parameterized with  $(\Omega, \eta, \kappa) = (2.0 \times 10^{-3}, 8.0 \times 10^{-4}, 1.15 \times 10^{-4})$  implies  $\mathbf{S} = 0.512$  and  $\mathbf{D} = 1.50$ . As reported in Figure 7, this variation does not change our main result of an optimality of the zero-tax-rate on consumption good.

As another robustness check, we consider the case in which the cash withdrawing cost for

Figure 7: Welfare: fixed cash handling cost for sellers



a buyer is fixed and the cash handling cost for a seller is proportional. That is, withdrawing cash incurs a fixed cost  $\bar{\eta}$  for a buyer and cash handling cost for a seller increases with the value of cash transaction at a rate of  $\tilde{\kappa}$ .<sup>8</sup> In computing a steady state for this case, we again choose  $(\bar{\eta}, \tilde{\kappa})$  to fit the model to the data. The model parameterized with  $(\Omega, \bar{\eta}, \tilde{\kappa}) = (3.5 \times 10^{-3}, 1.01 \times 10^{-3}, 1.0 \times 10^{-3})$  implies  $\mathbf{S} = 0.568$  and  $\mathbf{D} = 1.56$ . As reported in Figure 8, our main result of an optimality of the zero-tax-rate on consumption good is still immune to this variation.

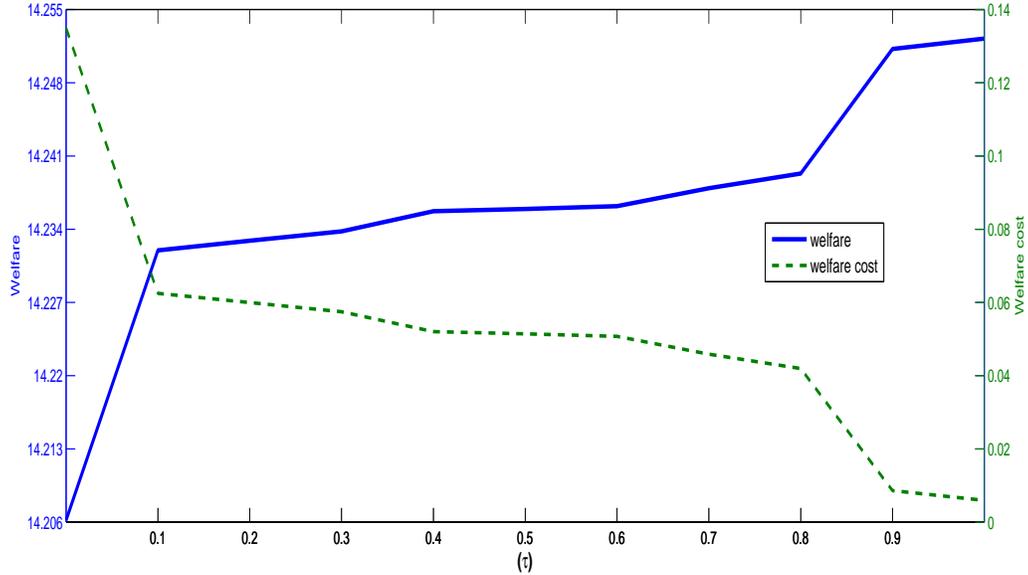
## 5. Concluding Remarks

In this paper, we have explored an optimal allocation of the social cost for electronic payment system by incorporating a standard Ramsey taxation approach into an off-the-shelf matching

---

<sup>8</sup>Notice that if  $\tilde{\eta} \geq (\bar{\eta}/N)$ , as in a benchmark model, all agents deposit their money into checking accounts at the beginning of each period.

Figure 8: Welfare: fixed cash withdrawing cost+proportional cash handling cost



model of money. Our results suggest that the economy of scale in the use of an electronic payment technology is enhanced with the tax rate on labor. This then decreases not only per transaction cost of electronic payment and cash withdrawing cost, but also welfare loss due to the distortionary consumption taxation. As a result, the zero tax rate on consumption good turns out to be optimal in allocating the social cost for electronic payment technology.

It is worthwhile to note that we have discussed an allocation method of the social cost for electronic payment system exclusively and industrial organization does not enter into our analysis. For instance, we do not deal with issue on how to impose fees on merchants and consumers by a profit maximizing card issuer. We also assume that the terms of trade are determined via a non-competitive pricing mechanism not a competitive pricing mechanism. We leave to future research the extensions or variations of our model from the point of industrial organization.

## References

- Aiyagari, R., Braun, R., Eckstein, Z., 1998. Transaction Services, Inflation, and Welfare. *Journal of Political Economy* 106, 1274-1301.
- Aiyagari, R., Wallace, N., Wright, R., 1996. Coexistence of Money and Interest-Bearing Securities. *Journal of Monetary Economics* 37, 397-419.
- Aliprantis, C.D., Camera, G., Puzzello, D., 2007. Anonymous Markets and Monetary Trading. *Journal of Monetary Economics* 54, 1905-1928.
- Berentsen, A., Camera, G., Waller, C., 2005. The Distribution of Money Balances and the Nonneutrality of Money. *International Economic Review* 46, 465-487.
- Camera, G., Corbae, D., 1999. Money and Price Dispersion. *International Economic Review* 40, 985-1008.
- Corbae, D., Temzelides, T., Wright, R., 2003. Directed Search and Monetary Exchange. *Econometrica* 71, 731-756.
- Green, E., Zhou, R., 1998. A Rudimentary Random-Matching Model with Divisible Money and Prices. *Journal of Economic Theory* 81, 252-271.
- Green, E., Zhou, R., 2002. Dynamic Monetary Equilibrium in a Random Matching Economy. *Econometrica* 70, 929-969.
- Hayashi, F., Keeton, W., 2012. Measuring the Costs of Retail Payment Methods. *Economic Review* Second Quarter 2012, 37-77. Federal Reserve Bank of Kansas City.
- Kocherlakota, N., 1998. Money is Memory. *Journal of Economic Theory* 81, 232-251.
- Lee, M., Wallace, N., Zhu, T., 2005. Modeling Denomination Structures. *Econometrica* 73, 949-960.
- Li, Y., 2011. Currency and Checking Deposits as Means of Payment. *Review of Economic Dynamics* 14, 403-417.
- Molico, M., 2006. The Distribution of Money and Prices in Search Equilibrium. *International Economic Review* 47, 701-722.
- Shi, S., 1995. Money and Prices: A Model of Search and Bargaining. *Journal of Economic Theory* 67, 467-498.
- Shy, Oz. 2012. Trends in Consumer Payments. Consumer Payments Research Center, Federal Reserve Bank of Boston.
- Stavins, J., 2012. Consumer Payment Cards: Lessons from Consumer Surveys. Consumer Payments Research Center, Federal Reserve Bank of Boston.

Trejos, A., Wright, R., 1995. Search, Bargaining, Money and Prices. *Journal of Political Economy* 103, 118-141.

Wallace, N., 2001. Whither Monetary Economics? *International Economic Review* 42, 847-870.

Zhu, T., 2003. Existence of Monetary Steady States in a Matching Model: Indivisible Money. *Journal of Economic Theory* 112, 307-324.